

Lista3: Produto Cartesiano e Relações

- 1) Sejam os conjuntos $A = (-3, 2]$, $B = (-\infty - 2] \cup [2, \infty)$ e $C = \{1, 2, 3, 4\}$.
- (a) Represente graficamente $A \times B$ e $A \times C$.
- (b) Seja S a relação em \mathbb{R} definida por $(x, y) \in S \Leftrightarrow (x \geq 0) \wedge (y \in B)$. Determine o domínio e a imagem de S e represente-a graficamente.
- 2) Sejam os conjuntos $A = \{1, \sqrt{2}, 2\}$ e $B = \{2^n : n \in \mathbb{Z}, 1 \leq n \leq 4\}$.
- (a) Determine $A \times B$;
- (b) Determine o domínio e a imagem da relação R de A em B definida por:
 $x R y$ se e somente se $y = x^2$

3) Sejam os conjuntos $A = \{1, \sqrt{2}, 2, 3\}$ e $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & \sqrt{2} \\ -1 & 0 \end{bmatrix} \right\}$

- (a) Determine a cardinalidade de $A \times B$ e apresente dois elementos deste conjunto;
- (b) Determine o domínio e a imagem da relação R de A em B definida por:
 $x R Y$ se e somente se x é o determinante da matriz Y .

Lembre que $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

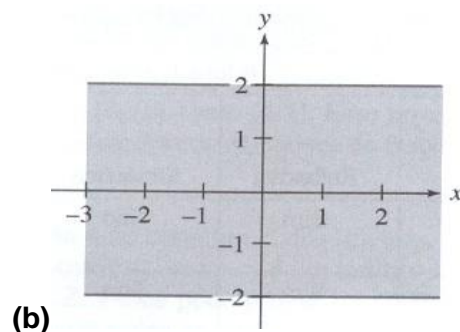
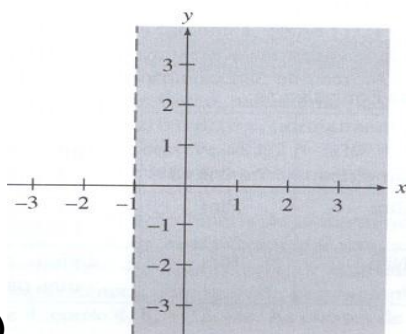
- 4) Sejam as relações R_1, R_2, R_3 definidas por:

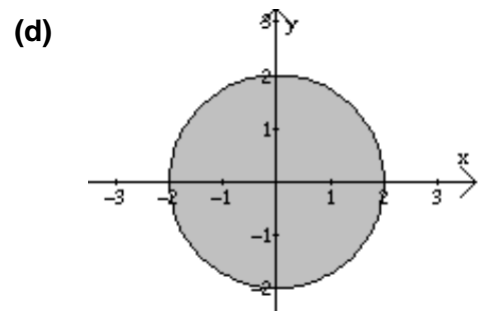
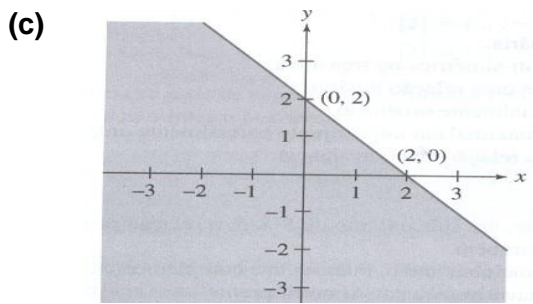
$R_1 = \{(x, y) \in \mathbb{R}^2 / y > x\}$ $R_2 = \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 \leq 4\}$ $R_3 = \{(x, y) \in \mathbb{R}^2 / x = y \vee x = -y\}$

Represente graficamente e determine o domínio e a imagem das relações:

- (a) R_1 (b) R_2 (c) R_3 (d) $R_1 \cap R_3$
 (e) $R_1 - R_2$ (f) $R_2 \cup R_3$ (g) $R_1 \cap R_2$

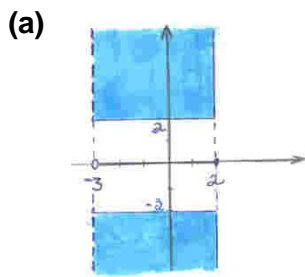
- 5) Para cada uma das figuras a seguir, diga qual a relação binária que descreve a área sombreada.



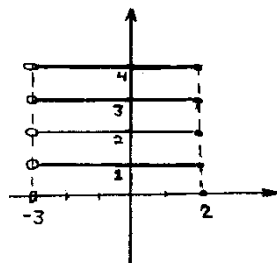


Respostas:

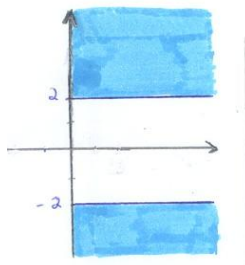
1)



$A \times B$



$A \times C$



(b)

S

2)

(a)

$$A \times B = \{(1, 2), (1, 4), (1, 8), (1, 16), (\sqrt{2}, 2), (\sqrt{2}, 4), (\sqrt{2}, 8), (\sqrt{2}, 16), (2, 2), (2, 4), (2, 8), (2, 16)\}$$

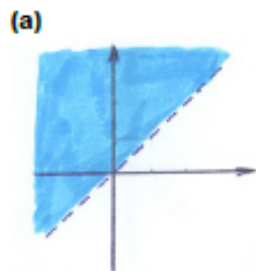
(b) $Dom R = \{\sqrt{2}, 2\}$ e $Im R = \{2, 4\}$

3)

(a) $|A \times B| = 12$, $\left(1, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$ e $\left(2, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right)$ são elementos de $A \times B$.

(b) $Dom R = \{1, \sqrt{2}, 3\}$ e $Im R = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & \sqrt{2} \\ -1 & 0 \end{bmatrix} \right\} = B$

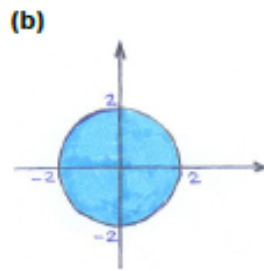
4)



$$Dom R_1 = \mathbb{R}$$

$$Im R_1 = \mathbb{R}$$

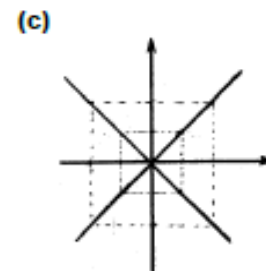
(d)



$$Dom R_2 = [-2, 2]$$

$$Im R_2 = [-2, 2]$$

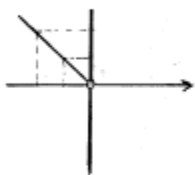
(e)



$$Dom R_3 = \mathbb{R}$$

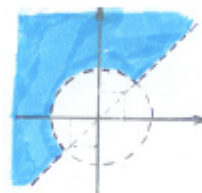
$$Im R_3 = \mathbb{R}$$

(f)



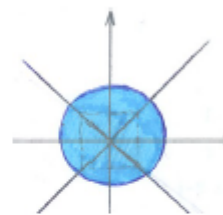
$$Dom(R_1 \cap R_3) = (-\infty, 0)$$

$$Im(R_1 \cap R_3) = (0, +\infty)$$



$$Dom(R_1 \setminus R_2) = \mathbb{R}$$

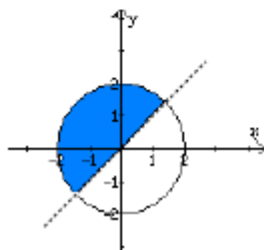
$$Im(R_1 \setminus R_2) = \mathbb{R}$$



$$Dom(R_2 \cup R_3) = \mathbb{R}$$

$$Im(R_2 \cup R_3) = \mathbb{R}$$

(g)



$$Dom(R_1 \cap R_2) = [-2, \sqrt{2}]$$

$$Im(R_1 \cap R_2) = (-\sqrt{2}, 2]$$

5)

(a) $R = \{(x, y) \in \mathbb{R}^2 \mid x > -1\}$

(b) $R = \{(x, y) \in \mathbb{R}^2 \mid -2 \leq y \leq 2\}$

(c) $R = \{(x, y) \in \mathbb{R}^2 \mid y \leq 2 - x\}$

(d) $R = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$